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Batalin-Tyutin Quantization of the Chern-Simons-Proca Theory

Ei-Byung Park, Yong-Wan Kim, Young-Jai Park, and Yongduk Kim
Department of Physics and Basic Science Research Institute
Sogang University, C.P.O. Box 1142, Seoul 100-611, Korea

Won Tae Kim
Center for Theoretical Physics and Department of Physics
Seoul National University, Seoul 151-742, Korea

ABSTRACT

We quantize the Chern-Simons-Proca theory in three dimensions by using the Batalin-Tyutin Hamiltonian method, which systematically embeds second class constraint system into first class by introducing new fields in the extended phase space. As results, we obtain simultaneously the Stückelberg scalar term, which is needed to cancel the gauge anomaly due to the mass term, and the new type of Wess-Zumino action, which is irrelevant to the gauge symmetry. We also investigate the infrared property of the Chern-Simons-Proca theory by using the Batalin-Tyutin formalism comparing with the symplectic formalism. As a result, we observe that the resulting theory is precisely the gauge invariant Chern-Simons-Proca quantum mechanical version of this theory.

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1 Introduction

The Dirac method has been widely used in the Hamiltonian formalism [1] to quantize second class constraint systems. However, since the resulting Dirac brackets are generally field-dependent and nonlocal, and have a serious ordering problem between field operators, these are under unfavorable circumstances in finding canonically conjugate pairs. On the other hand, the quantization of first class constraint systems [2,3] has been well appreciated in a gauge invariant manner preserving Becci-Rouet-Stora-Tyutin (BRST) symmetry [4,5]. This formalism has been extensively studied by Batalin, Fradkin, and Tyutin [6,7] in canonical formalism, and applied to various models [8-10] obtaining the Wess-Zumino (WZ) action [11,12]. Recently, Banerjee [13] has applied the Batalin-Tyutin (BT) Hamiltonian method [7] to the abelian Chern-Simons (CS) field theory [14-16]. As a result, he has obtained the new type of an abelian WZ action, which cannot be obtained in the usual path-integral framework. Very recently, we have quantized the nonabelian case by generalizing this BT formalism [17]. As shown in these works, the nature of second class constraint algebra in the original theories originates from the symplectic structure of CS term, not due to the local gauge symmetry breaking. Banerjee, Ghosh, and Banerjee [18] have also considered a massive Maxwell theory. As a result, the extra field in this approach has identified with the Stückelberg scalar. We have also quantized the abelian self-dual massive theory by using this formalism, which interestingly produces both the Stückelberg scalar and the new type of WZ [19]. There are some other interesting examples in this approach [20].

On the other hand, three-dimensional Chern-Simons gauge theories have been attracting much attention because these play an important role in the present development of the quantum Hall effect [21] and the string theory [22]. The quantum mechanical version of the CS field theory has been studied by Jackiw and collaborators through the phase space reductive limiting procedure [23]. Recently, Baxter [24] has described a simple (2+1)-dimensional system which allow in principle as the experimental verification of the CS feature by introducing the Röntgen energy term [25].

In the present paper, we shall apply the BT Hamiltonian method [7] to the Chern-Simons-Proca (CSP) theory [26] revealing both the Stückelberg effect [27] and the CS effect [13,17]. In Sec. 2, we apply the BT formalism to the CSP theory in three dimensions which is gauge non-invariant. By identifying the new fields ρ and λ with the Stückelberg scalar and the WZ scalar, respectively, we obtain simultaneously the Stückelberg scalar term related to the explicit gauge-symmetry-breaking mass term and the new type of WZ action related to the symplectic structure of the CS term. In Sec. 3, we also investigate the quantum mechanical version of the CSP theory by using the BT formalism comparing with the symplectic formalism [29], which is the improved version of the Dirac method, and, in particular, very effective for the first-order Lagrangian. As a result, we observe that the resulting theory is just the gauge invariant CSP quantum mechanical model.

2 The Chern-Simons-Proca Theory

We consider the abelian CSP model [26]

$$S_{CSP} = \int d^3x \; [-\frac{1}{2}\kappa\epsilon_{\mu\nu\rho}A^\mu\partial^\nu A^\rho + \frac{1}{2}m^2 A^\mu A_\mu] \tag{1}$$

by using the BT formalism. Note that this action has an explicit mass term, which breaks the gauge symmetry as the case of the Proca model [18], and also the CS term, which has a different origin of the second class constraint system. Consequently, this action represents the second class constraint system combined with two effects, which can be easily confirmed by the standard Dirac analysis [1]. There are three primary constraints,

$$\begin{aligned} \Omega_0 &\equiv \pi_0 \approx 0, \\ \Omega_i &\equiv \pi_i + \frac{1}{2}\kappa\epsilon_{ij}A^j \approx 0, \qquad (i = 1, 2), \end{aligned} \tag{2}$$

and one secondary constraint,

$$\omega_3 \equiv m^2 A^0 - \kappa\epsilon_{ij}\partial^i A^j \approx 0, \tag{3}$$

which is obtained by conserving Ω_0 with the total Hamiltonian,

$$H_T = H_c + \int d^2x[u^0\Omega_0 + u^i\Omega_i], \tag{4}$$

where H_c is the canonical Hamiltonian,

$$H_c = \int d^2x \left[\kappa\epsilon_{ij}A^0\partial^i A^j + \frac{1}{2}m^2\{(A^i)^2 - (A^0)^2\}\right], \tag{5}$$

and u^0, u^i are Lagrange multipliers. No further constraints are generated via this procedure. We find that all constraints are fully second class. In order to carry out the simple algebraic manipulation, it is, however, essential to redefine ω_3 by using Ω_i as follows

$$\begin{aligned} \Omega_3 &\equiv \omega_3 + \partial^i\Omega_i \\ &= \partial^i\pi_i - \frac{1}{2}\kappa\epsilon_{ij}\partial^i A^j + m^2 A^0, \end{aligned} \tag{6}$$

although the redefined constraints $\Omega_\alpha(\alpha = 0, 1, 2, 3)$ are still completely second class. Otherwise, one will have a complicated constraint algebra including the derivative terms, which is difficult to handle. Then, the modified constraint algebra is given by

$$\begin{aligned} \Delta_{\alpha\beta}(x, y) &\equiv \{\Omega_\alpha(x), \Omega_\beta(y)\} \\ &= \begin{pmatrix} 0 & 0 & 0 & -m^2 \\ 0 & 0 & \kappa & 0 \\ 0 & -\kappa & 0 & 0 \\ m^2 & 0 & 0 & 0 \end{pmatrix} \delta^2(x - y), \end{aligned} \tag{7}$$

which reveals the simple second class nature of the constraints Ω_α.

In order to convert this system into first class, the first objective is to transform Ω_α into the first class by extending a phase space. Following the BT approach [7], we introduce new auxiliary fields Φ^α, and assume that the Poisson algebra of the new fields is given by

$$\{\Phi^\alpha(x), \Phi^\beta(y)\} = \omega^{\alpha\beta}(x,y), \tag{8}$$

where $\omega^{\alpha\beta}$ is an antisymmetric matrix. Then, the modified constraint in the extended phase space is given by

$$\tilde{\Omega}_\alpha(\pi_\mu, A^\mu, \Phi^\beta) = \Omega_\alpha + \sum_{n=1}^{\infty} \Omega_\alpha^{(n)}; \qquad \Omega_\alpha^{(n)} \sim (\Phi^\beta)^n, \tag{9}$$

satisfying the boundary condition, $\tilde{\Omega}_\alpha(\pi_\mu, A^\mu, 0) = \Omega_\alpha$. The first order correction term in the infinite series [7] is given by

$$\Omega_\alpha^{(1)}(x) = \int d^2y X_{\alpha\beta}(x,y)\Phi^\beta(y), \tag{10}$$

and the first class constraint algebra of $\tilde{\Omega}_\alpha$ requires the condition as follows

$$\triangle_{\alpha\beta}(x,y) + \int d^2w \; d^2z \; X_{\alpha\gamma}(x,w)\omega^{\gamma\delta}(w,z)X_{\delta\beta}(z,y) = 0. \tag{11}$$

Among the solutions satisfying with the conditions (8) and (11), we take a simple solution as follows

$$\begin{aligned} \omega^{\alpha\beta}(x,y) &= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \delta^2(x-y), \\ X_{\alpha\beta}(x,y) &= \begin{pmatrix} m & 0 & 0 & 0 \\ 0 & \sqrt{\kappa} & 0 & 0 \\ 0 & 0 & \sqrt{\kappa} & 0 \\ 0 & 0 & 0 & m \end{pmatrix} \delta^2(x-y). \end{aligned} \tag{12}$$

There is an arbitrariness in choosing $\omega^{\alpha\beta}$, which would naturally be manifested in Eq. (12). This just corresponds to the canonical transformations in the extended phase space. However, as has been shown in other calculations [13,17], this choice of Eq. (12) gives the remarkable algebraic simplification.

Using Eqs. (9) and (12), the new set of constraints is found to be

$$\begin{aligned} \tilde{\Omega}_0 &= \Omega_0 + m\Phi^0, \\ \tilde{\Omega}_i &= \Omega_i + \sqrt{\kappa}\Phi^i, \qquad (i = 1, 2), \\ \tilde{\Omega}_3 &= \Omega_3 + m\Phi^3, \end{aligned} \tag{13}$$

which are strongly involutive,

$$\{\tilde{\Omega}_\alpha, \tilde{\Omega}_\beta\} = 0. \tag{14}$$

As a result, we have fully first class constraints in the extended phase space by applying the BT formalism systematically. Observe further that only the $\Omega_\alpha^{(1)}$ contributes in the series (9) defining the first class constraint. All higher order terms given by Eq. (9) vanish as a consequence of the choice Eq. (12). Recall the Φ^α are the new variables satisfying the algebra (8) with $\omega^{\alpha\beta}$ given by Eq. (12).

The next step is to obtain the involutive Hamiltonian, which naturally generates the secondary Gauss constraint in the extended phase space. It is given by the infinite series [7],

$$\tilde{H} = H_c + \sum_{n=1}^{\infty} H^{(n)}; \qquad H^{(n)} \sim (\Phi^\alpha)^n, \tag{15}$$

satisfying the initial condition, $\tilde{H}(\pi_\mu, A^\mu, 0) = H_c$. The general solution [7] for the involution of $\tilde{H}$ is given by

$$H^{(n)} = -\frac{1}{n}\int d^2x d^2y d^2z \ \Phi^\alpha(x)\omega_{\alpha\beta}(x,y)X^{\beta\gamma}(y,z)G_\gamma^{(n-1)}(z), \quad (n \geq 1), \tag{16}$$

where the generating functionals $G_\alpha^{(n)}$ are given by

$$\begin{aligned} G_\alpha^{(0)} &= \{\Omega_\alpha^{(0)}, H_c\}, \\ G_\alpha^{(n)} &= \{\Omega_\alpha^{(0)}, H^{(n)}\}_{\mathcal{O}} + \{\Omega_\alpha^{(1)}, H^{(n-1)}\}_{\mathcal{O}} \quad (n \geq 1), \end{aligned} \tag{17}$$

where the symbol $\mathcal{O}$ in Eq. (17) represents that the Poisson brackets are calculated among the original variables, *i.e.*, $\mathcal{O} = (\pi_\mu, A^\mu)$. Here, $\omega_{\alpha\beta}$ and $X^{\alpha\beta}$ are the inverse matrices of $\omega^{\alpha\beta}$ and $X_{\alpha\beta}$ respectively. Explicit calculations of $G_\alpha^{(0)}$ yield,

$$\begin{aligned} G_0^{(0)} &= m^2 A^0 - \kappa\epsilon_{ij}\partial^i A^j, \\ G_i^{(0)} &= -m^2 A^i - \kappa\epsilon_{ij}\partial^j A^0, \\ G_3^{(0)} &= m^2 \partial_i A^i, \end{aligned} \tag{18}$$

which are substituted in Eq. (16) to obtain $H^{(1)}$,

$$H^{(1)} = \int d^2x \left[m\Phi^0\partial_i A^i + \frac{m^2}{\sqrt{\kappa}}\epsilon_{ij}\Phi^i A^j + \sqrt{\kappa}\Phi^i\partial_i A^0 - \Phi^3(mA^0 - \frac{\kappa}{m}\epsilon_{ij}\partial^i A^j)\right]. \tag{19}$$

This is inserted back in Eq. (17) to deduce $G_\alpha^{(1)}$ as follows

$$\begin{aligned} G_0^{(1)} &= \sqrt{\kappa}\partial_i\Phi^i + m\Phi^3, \\ G_i^{(1)} &= m\partial_i\Phi^0 + \frac{m^2}{\sqrt{\kappa}}\epsilon_{ij}\Phi^j - \frac{\kappa}{m}\epsilon_{ij}\partial^j\Phi^3, \\ G_3^{(1)} &= m\partial_i\partial^i\Phi^0 + \frac{m^2}{\sqrt{\kappa}}\epsilon_{ij}\partial^i\Phi^j, \end{aligned} \tag{20}$$

which then yield $H^{(2)}$ from Eq. (16),

$$H^{(2)} = \int d^2x \left[-\frac{1}{2}\partial_i\Phi^0\partial^i\Phi^0 + \frac{m}{\sqrt{\kappa}}\Phi^0\epsilon_{ij}\partial^i\Phi^j + \frac{m^2}{2\kappa}\Phi^i\Phi^i - (\frac{\sqrt{\kappa}}{m}\partial_i\Phi^i + \frac{1}{2}\Phi^3)\Phi^3\right]. \tag{21}$$

Since $G_\alpha^{(n)} = 0 \quad (n \geq 2)$, the final expression for the involutive Hamiltonian after the $n = 2$ finite truncations is given by

$$\tilde{H} = H_c + H^{(1)} + H^{(2)}, \tag{22}$$

which is strongly involutive,

$$\{\tilde{\Omega}_\alpha, \tilde{H}\} = 0. \tag{23}$$

According to the usual BT formalism, this formally completes the operatorial conversion of the original second class system with Hamiltonian H_c and constraints Ω_α into the first class with Hamiltonian $\tilde{H}$ and constraints $\tilde{\Omega}_\alpha$.

However, before performing the momentum integrations to obtain the partition function in the configuration space, it seems appropriate to comment on the strongly involutive Hamiltonian. If we directly use this Hamiltonian, we can not naturally generate the first class Gauss' law constraint $\tilde{\Omega}_3$ from the time evolution of the primary constraint $\tilde{\Omega}_0$, which is the first class. Therefore, in order to avoid this problem, we use the equivalent first class Hamiltonian without any loss of generality, which only differs from the involutive Hamiltonian (22) by adding a term proportional to the first class constraint $\tilde{\Omega}_3$ as follows

$$\tilde{H}' = \tilde{H} + \frac{1}{m}\Phi^3\tilde{\Omega}_3. \tag{24}$$

Then, this desired Hamiltonian $\tilde{H}'$ automatically generates the Gauss' law constraint such that $\{\tilde{\Omega}_0, \tilde{H}'\} = \tilde{\Omega}_3$. Note that when we act this modified Hamiltonian on physical states, the difference with $\tilde{H}$ is trivial because such states are annihilated by the first class constraint. Similarly, the equations of motion for observable (*i.e.* gauge invariant variables) will also be unaffected by this difference since $\tilde{\Omega}_3$ can be regarded as the generator of the gauge transformations.

We now derive the Lagrangian, which will include both the Stückelberg effect and the CS effect, corresponding to the Hamiltonian (24). The first step is to identify the new variables Φ^α as canonically conjugate pairs in the Hamiltonian formalism as follows

$$\Phi^\alpha \equiv (m\rho, \frac{1}{\sqrt{\kappa}}\pi_\lambda, \sqrt{\kappa}\lambda, \frac{1}{m}\pi_\rho) \tag{25}$$

satisfying Eqs. (8) and (12). The starting phase space partition function is then given by the Faddeev formula [28],

$$Z = \int \mathcal{D}A^\mu \mathcal{D}\pi_\mu \mathcal{D}\lambda \mathcal{D}\pi_\lambda \mathcal{D}\rho \mathcal{D}\pi_\rho \prod_{\alpha,\beta=0}^{3} \delta(\tilde{\Omega}_\alpha)\delta(\Gamma_\beta) det \mid \{\tilde{\Omega}_\alpha, \Gamma_\beta\} \mid e^{iS'}, \tag{26}$$

where

$$S' = \int d^3x \left(\pi_\mu \dot{A}^\mu + \pi_\lambda \dot{\lambda} + \pi_\rho \dot{\rho} - \tilde{\mathcal{H}}'\right) \tag{27}$$

with the Hamiltonian density $\tilde{\mathcal{H}}'$ corresponding to $\tilde{H}'$, which is now expressed in terms of $\{\rho, \pi_\rho, \lambda, \pi_\lambda\}$ instead of Φ^α. The gauge fixing conditions Γ_α may be assumed to be

independent of the momenta so that these are considered as the Faddeev-Popov type gauge conditions [28].

Next, we perform the momentum integrations to obtain the configuration space partition function. The π_0, π_1, and π_2 integrations are trivially performed by exploiting the delta functions $\delta(\tilde{\Omega}_0) = \delta(\pi_0 + m^2\rho)$, $\delta(\tilde{\Omega}_1) = \delta(\pi_1 + \frac{\kappa}{2}A^2 + \pi_\lambda)$, and $\delta(\tilde{\Omega}_2) = \delta(\pi_2 - \frac{\kappa}{2}A^1 + \kappa\lambda)$, respectively. After exponentiating the remaining delta function $\delta(\tilde{\Omega}_3) = \delta(-\kappa\epsilon_{ij}\partial^i A^j + \partial_1\pi_\lambda + \kappa\partial_2\lambda + m^2A^0 + \pi_\rho)$ with Fourier variable ξ as $\delta(\tilde{\Omega}_3) = \int \mathcal{D}\xi e^{-i\int d^3x\xi\tilde{\Omega}_3}$ and transforming $A^0 \to A^0 + \xi$, we obtain the action as follows

$$
\begin{aligned}
S &= \int d^3x \, \{-\frac{1}{2}\kappa\epsilon_{\mu\nu\rho}A^\mu\partial^\nu A^\rho + \frac{1}{2}m^2A^\mu A_\mu \\
&+ \rho[-m^2(\dot{A}^0 + \dot{\xi}) - m^2\partial_i A^i - \frac{1}{2}m^2\partial_i\partial^i\rho + m^2\partial_1\lambda - \frac{m^2}{\kappa}\partial_2\pi_\lambda] \\
&+ \pi_\rho[\dot{\rho} - \frac{1}{2m^2}\pi_\rho - \xi] + \lambda[-\kappa\dot{A}^2 + m^2A^1 - \kappa\partial_2A^0 - \frac{1}{2}m^2\lambda] \\
&+ \pi_\lambda[\dot{\lambda} - \dot{A}^1 - \frac{m^2}{\kappa}A^2 + \partial^1A^0 - \frac{1}{2}\pi_\lambda] - \frac{1}{2}m^2\xi^2\},
\end{aligned}
\tag{28}
$$

where the overdot means the time derivative, and the corresponding measure is given by

$$
[\mathcal{D}\mu] = \mathcal{D}A^\mu\mathcal{D}\lambda\mathcal{D}\pi_\lambda\mathcal{D}\rho\mathcal{D}\pi_\rho\mathcal{D}\xi \prod_{\beta=0}^{3} \delta(\Gamma_\beta[A^0 + \xi, A^i, \lambda, \rho]) det \mid \{\tilde{\Omega}_\alpha, \Gamma_\beta\} \mid, \tag{29}
$$

where $A^0 \to A^0 + \xi$ transformation is naturally understood in the gauge fixing condition Γ_β.

Note that the original theory is easily reproduced in one line, *i.e.*, if we choose the unitary gauge

$$
\Gamma_\alpha = (\rho, \pi_\lambda, \lambda, \pi_\rho), \tag{30}
$$

and integrate over ξ. Then, one can easily realize that the new fields Φ^α are nothing but the gauge degrees of freedom, which can be removed by utilizing the gauge symmetry.

Now, we perform the Gaussian integration over π_ρ. Then all terms including ξ in the action are canceled out, and integrating over π_λ the resultant action is finally obtained as follows

$$
\begin{aligned}
S &= S_{St} + S_{NWZ} + S_B \;; \\
S_{St} &= \int d^3x\{-\frac{1}{2}\kappa\epsilon_{\mu\nu\rho}A^\mu\partial^\nu A^\rho + \frac{1}{2}m^2(A_\mu + \partial_\mu\rho)^2\}, \\
S_{NWZ} &= \int d^3x\{\frac{\kappa^2}{2m^2}[\dot{\lambda} + F_{01} + \frac{m^2}{\kappa}(A_2 + \partial_2\rho)]^2 \\
&\qquad + \kappa\lambda[F_{02} - \frac{m^2}{\kappa}(A_1 + \partial_1\rho) - \frac{m^2}{2\kappa}\lambda]\}, \\
S_B &= -\int d^3x \; \partial_\mu(m^2\rho A^\mu)
\end{aligned}
\tag{31}
$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Note that S_{St} is an expected Stückelberg scalar term, which is needed to cancel the gauge anomaly due to the explicit gauge-symmetry-breaking mass term [18], S_{NWZ} is the new type of WZ term due to the symplectic structure of the CS term [13,19], which is irrelevant to the gauge symmetry, and S_B is the boundary term, which is also needed to make the second class system into the first class. The corresponding Liouville measure just comprises the configuration space variables as follows

$$[\mathcal{D}\mu] = \mathcal{D}A^\mu \mathcal{D}\lambda \mathcal{D}\rho \mathcal{D}\xi \prod_{\beta=0}^{3} \delta(\Gamma_\beta[A^0 + \xi, A^i, \lambda, \rho]) det \mid \{\tilde{\Omega}_\alpha, \Gamma_\beta\} \mid . \quad (32)$$

This action S is invariant up to the total divergence under the gauge transformations as $\delta A_\mu = \partial_\mu \Lambda$, $\delta\rho = -\Lambda$, and $\delta\lambda = 0$.

Note that starting from the action (31) with the boundary term S_B, we can easily reproduce the same set of all first class constraints $\tilde{\Omega}_\alpha$, and the Hamiltonian such that

$$\begin{aligned} H = H_c \quad &+ \quad \int d^2x \ \ [\pi_\lambda \partial_1 A_0 - \frac{m^2}{\kappa} \pi_\lambda A_2 + m^2 \rho \partial_i A^i + \kappa\lambda\partial_2 A_0 + m^2 \lambda A_1 \\ &+ \quad \frac{m^2}{2\kappa^2}\pi_\lambda^2 - \frac{m^2}{\kappa}\pi_\lambda \partial_2 \rho - \frac{1}{2} m^2 \partial_i \rho \partial^i \rho + m^2 \lambda \partial_1 \rho + \frac{1}{2} m^2 \lambda^2 + \frac{1}{2m^2}\pi_\rho^2]. \quad (33) \end{aligned}$$

Then, if we add a term proportional to the constraint $\tilde{\Omega}_3$, *i.e.*, $\frac{1}{m^2}\pi_\rho \tilde{\Omega}_3$, which is trivial when acting on the physical Hilbert space, to the above Hamiltonian (33), we can obtain the original involutive Hamiltonian (22). Furthermore, this difference is also trivial in the construction of the functional integral because the constraint $\tilde{\Omega}_3$ is strongly implemented by the delta function $\delta(\tilde{\Omega}_3)$ in Eq. (26). Therefore, we have shown that the constraints and the Hamiltonian following from the action (31) are effectively equivalent to the original Hamiltonian embedding structure. As results, through the BT quantization procedure, we have found that the Stückelberg scalar ρ is naturally introduced in the mass term, and this ρ as well as the WZ scalar λ is also included in the new type of WZ action.

We also note that if we ignore the boundary term S_B in this action, we cannot directly obtain the involutive first class Hamiltonian as the case of the Proca theory explained in Ref. [18] because this boundary term plays the important role in this procedure.

Finally, note that in the trivial limit $\kappa \to m$, the action (31) exactly reduces to the self-dual massive theory having all the first class constraints, which has recently been derived in Ref. [19]. The limit $m \to 0$ is non-trivial because the action has the m^{-2} term. In fact, we can easily find that the auxiliary field π_λ is not well-defined in the action for the case of this limit because the π_λ contains the m^{-2} term, *i.e.*, $\pi_\lambda = \frac{\kappa^2}{m^2}(\dot{\lambda} + F_{01}) + \kappa(A_2 + \partial_2\rho)$. Therefore, we have to pay attention to when the momentum integrations are performed in the below of Eq. (28). Avoiding this situation, if we simultaneously take the limit $(\dot{\lambda} + F_{01}) \to 0$ with the $m \to 0$, we obtain the delta function $\delta(\dot{\lambda} + F_{01})$ in the measure part when we perform the momentum integration over π_λ resulting to the case of the pure CS action. Then, one can finally

find that the CSP theory exactly reduces to the pure CS case having all the first class constraints [19].

3 The Chern-Simons-Proca Quantum Mechanics

3.1 The Symplectic Quantization of the CSP Quantum Mechanics

Let us briefly discuss the symplectic quantization, which is very effective for the first-order Lagrangian [29], of the CSP quantum mechanics. We start the following infrared limit action [23], which is already first-order, of the CSP theory with the Coulomb gauge

$$S_o = \int dt L_o = \int dt \left[\frac{1}{2}\kappa\epsilon_{ij}q^i\dot{q}^j - \frac{1}{2}mq^iq^i\right], \tag{34}$$

where $\epsilon_{12} = \epsilon^{12} = 1$. Following the symplectic formalism, we first rewrite the action as follows

$$S_o = \int dt \left[-\frac{\kappa}{2}q^2\dot{q}^1 + \frac{\kappa}{2}q^1\dot{q}^2 - H^{(0)}\right], \tag{35}$$

where $H^{(0)} = \frac{m}{2}[(q^1)^2 + (q^2)^2]$ is the usual canonical Hamiltonian and the superscript denotes the number of iterations [29]. Note that since the action (35) is already first-ordered from the start, we do not need to introduce auxiliary fields such as conjugate momenta. Then, we set symplectic variables $\xi^{(0)i} = (q^1, q^2)$, and symplectically conjugated momenta $a_i^{(0)} = (-\frac{\kappa}{2}q^2, \frac{\kappa}{2}q^1)$.

Now, symplectic 2-form matrix f_{ij}, which consists of the essential part for finding the generalized brackets of the symplectic formalism, is obtained as follows

$$f_{ij}^{(0)} = \frac{\partial a_j^{(0)}}{\partial\xi^{(0)i}} - \frac{\partial a_i^{(0)}}{\partial\xi^{(0)j}} = \kappa\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{36}$$

having the antisymmetric property. This matrix is not singular, and thus has the inverse as follows

$$f^{(0)ij} = -\frac{1}{\kappa}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{37}$$

which gives the generalized symplectic brackets

$$\{q^i, q^j\} = -\frac{1}{\kappa}\epsilon^{ij}. \tag{38}$$

These are exactly same as the Dirac brackets [23] when we analyze the system through the usual Dirac's method.

It is appropriate to comment on the symplectic formalism [29]. In general, the symplectic 2-form matrix is not invertible at the first stage of iterations. Then, we can find

some zero modes, which are related to generate the constraints in the symplectic formulation, and incorporate them into the canonical sector with some auxiliary variables to find the nonvanishing symplectic 2-form matrix. If we can find the invertible symplectic 2-form matrix at the finite stage of iterations, the inverse of the matrix gives the generalized brackets, which are equivalent to the usual Dirac brackets. However, when we can not find the invertible matrix even at the infinite stages of iterations, we can say the system has a gauge symmetry and use the zero modes to obtain the concrete rules of transformations [29]. Especially in the case of the CSP quantum mechanical model, we have the symplectic 2-form matrix at the first stage of iterations. Thus the system has no constraints in the symplectic quantization formalism, while this system has second class constraints [23] in the standard Dirac formalism as follows

$$\Omega_i \equiv p_i + \frac{1}{2}\kappa\epsilon_{ij}q^j \approx 0 \qquad (i = 1, 2), \tag{39}$$

where $p_i = \frac{\partial L_o}{\partial \dot{q}^i} = -\frac{1}{2}\epsilon_{ij}q^j$ are canonical momenta.

Now, using the generalized brackets, we can easily find the Hamilton equations as follows

$$\dot{q}^i = \{q^i, H^{(0)}\} = -\frac{m}{\kappa}\epsilon^{ij}q^j. \tag{40}$$

These equations give a single simple harmonic oscillator

$$\dot{\gamma}(t) \equiv \dot{q}^1 + i\dot{q}^2 = i\omega\gamma(t), \tag{41}$$

with the frequency $\omega = \frac{m}{\kappa}$ as usual. In the next section, we will show that starting the gauge non-invariant action (34), we can obtain the gauge invariant version describing the simple harmonic oscillator by the BT formalism.

3.2 The BT Quantization of the CSP Quantum Mechanics

Now let us analyze the action (34) in the BT quantization as in the previous Section 2. The first observed fact through the usual Dirac's procedure [1] is that the action represents a second class constraint system, *i.e.*, there are two primary constraints (39), and no further constraints are generated through the time evolution of these constraints with the total Hamiltonian,

$$H_T = H_c + u^i\Omega_i, \tag{42}$$

where H_c is the canonical Hamiltonian,

$$H_c = p_i\dot{q}^i - L_o = \frac{m}{2}q^iq^i, \tag{43}$$

and u^i are Lagrange multipliers. Then, the constraint algebra is given by

$$\begin{aligned}\Delta_{ij} &\equiv \{\Omega_i, \Omega_j\} = \kappa\epsilon_{ij} \\ &= \kappa\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\end{aligned} \tag{44}$$

which reveals the simple second class nature of the constraints Ω_i.

In order to convert this system into first class, the first objective is to transform Ω_i into the first class by extending the phase space. Following the BT approach [7], we introduce the matrix (8) as follows

$$\omega^{ij} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{45}$$

Then the other matrix X_{ij} in Eq. (10) is obtained by solving Eq. (11) with the Δ_{ij} given by Eq. (45),

$$X_{ij} = \sqrt{\kappa} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{46}$$

There is an arbitrariness in choosing ω^{ij}, which would naturally be manifested in Eq. (45) as explained in the Section 2.

Using Eqs. (9), (45) and (46), the new set of constraints is found to be

$$\tilde{\Omega}_i = \Omega_i + \sqrt{\kappa}\Phi^i, \tag{47}$$

which are strongly involutive,

$$\{\tilde{\Omega}_i, \tilde{\Omega}_j\} = 0. \tag{48}$$

The next step is to obtain the involutive Hamiltonian. The generating functions $G_i^{(n)}$ are obtained from Eq. (17). It is noteworthy that there are only two terms Ω_i and $\Omega_i^{(1)}$ in the expansion (47) due to the choice (45) and (46). Explicit calculations yield,

$$G_i^{(0)} = -mq^i, \tag{49}$$

which are substituted in Eq. (16) to obtain $H^{(1)}$,

$$H^{(1)} = \frac{m}{\sqrt{\kappa}}\epsilon_{ij}\Phi^i q^j. \tag{50}$$

This is inserted back in Eq. (17) to deduce $G_i^{(1)}$ as follows

$$G_i^{(1)} = \frac{m}{\sqrt{\kappa}}\epsilon_{ij}\Phi^j, \tag{51}$$

which then yield $H^{(2)}$ from Eq. (16),

$$H^{(2)} = \frac{m}{2\kappa}\Phi^i\Phi^i. \tag{52}$$

Since $G_i^{(n)} = 0 \;\; (n \geq 2)$, the final expression for the involutive Hamiltonian after the $n = 2$ finite truncations is given by

$$\tilde{H} = H_c + H^{(1)} + H^{(2)} = \frac{m}{2}[q^iq^i + \frac{2}{\sqrt{\kappa}}\epsilon_{ij}\Phi^i q^j + \frac{1}{\kappa}\Phi^i\Phi^i]. \tag{53}$$

which, by construction, is strongly involutive,

$$\{\tilde{\Omega}_i, \tilde{H}\} = 0. \tag{54}$$

This completes the operatorial conversion of the original second class system with Hamiltonian H_c and constraints Ω_i into the first class with Hamiltonian $\tilde{H}$ and constraints $\tilde{\Omega}_i$.

We now derive the gauge invariant Lagrangian corresponding to the Hamiltonian (53). The first step is to identify the new variables Φ^i as canonically conjugate pairs in the Hamiltonian formalism,

$$\Phi^\alpha \equiv (\frac{1}{\sqrt{\kappa}}P_X, \sqrt{\kappa}X), \tag{55}$$

satisfying Eqs. (8), (45) and (46). The starting phase space partition function is then given by the Faddeev formula,

$$Z = \int \mathcal{D}q^i \mathcal{D}p_i \mathcal{D}P_X \mathcal{D}X \prod_{i,j} \delta(\tilde{\Omega}_i)\delta(\Gamma_j) det \mid \{\tilde{\Omega}_i, \Gamma_j\} \mid e^{iS}, \tag{56}$$

where

$$S = \int dt \left(p_i \dot{q}^i + P_X \dot{X} - \tilde{H}\right). \tag{57}$$

As similar to the Section 2, the gauge fixing conditions Γ_i may be assumed to be independent of the momenta so that these are considered as the Faddeev-Popov type gauge conditions.

Next, we perform the momentum integrations to obtain the configuration space partition function. The p_1, and p_2 integrations are trivially performed by exploiting the delta functions $\delta(\tilde{\Omega}_1) = \delta(p_1 + \frac{\kappa}{2}q^2 + P_X)$, and $\delta(\tilde{\Omega}_2) = \delta(p_2 - \frac{\kappa}{2}q^1 + \kappa X)$, respectively. Then, we obtain the action as follows

$$\begin{aligned} S &= \int dt \, \{\frac{1}{2}\kappa\epsilon_{ij}q^i\dot{q}^j - \frac{1}{2}mq^iq^i \\ &+ X(-\kappa\dot{q}^2 + mq^1 - \frac{1}{2}mX) + P_X(-\dot{q}^1 + \dot{X} - \frac{m}{\kappa}q^2 - \frac{m}{2\kappa^2}P_X)\}, \end{aligned} \tag{58}$$

and the corresponding measure is given by

$$[\mathcal{D}\mu] = \mathcal{D}q^i \mathcal{D}X \mathcal{D}P_X \prod_{i,j} \delta(\Gamma_j) det \mid \{\tilde{\Omega}_i, \Gamma_j\} \mid . \tag{59}$$

Note that the original quantum mechanical model is also reproduced when we choose the unitary gauge such that $\Gamma_\alpha \equiv (X, P_X)$ as the case of the CSP theory.

Now, we perform the Gaussian integration over P_X. Then, the resultant action is finally obtained as follows

$$\begin{aligned} S &= S_o + S_{NWZ} \quad ; \\ S_{NWZ} &= \int dt \, \{\frac{\kappa^2}{2m}(\dot{q}^1 - \dot{X})^2 + \frac{m^2}{2}(q^2)^2 + \kappa q^2\dot{q}^1 + mXq^1 - \frac{1}{2}mX^2\}. \end{aligned} \tag{60}$$

We can rewrite the above action with the boundary term more compactly as follows

$$\begin{aligned} S &= \int dt \, \{\frac{\kappa^2}{2m}(\dot{q}^1 - \dot{X})^2 - \frac{m}{2}(q^1 - X)^2\} + S_B \ ; \\ S_B &= \int dt \, \frac{d}{dt}(-\kappa q^2 X + \frac{\kappa}{2} q^1 q^2). \end{aligned} \tag{61}$$

The corresponding Liouville measure just comprises the configuration space variables as follows

$$[\mathcal{D}\mu] = \mathcal{D}q^i \mathcal{D}X \prod_{i,j} \delta(\Gamma_j) det \mid \{\tilde{\Omega}_i, \Gamma_j\} \mid . \tag{62}$$

The action (61) is invariant up to the total divergence under the transformation $\delta q^1 = \epsilon(t)$ and $\delta X = \epsilon(t)$, which are just the gauge transformations of the CSP quantum mechanical model. Note that the q^2 variable has not appeared in Eq. (61) except the boundary term S_B. Furthermore, the action except the term S_B is just a usual harmonic oscillator having the frequency $\omega = \frac{m}{\kappa}$ as in the previous section when we define a quantity such that $\gamma(t) = q^1 - X$. Since $\delta\gamma(t)$ is invariant under the above transformations, it is a physical quantity. As a result, starting from the gauge non-invariant system (34), we obtain the gauge invariant version describing the harmonic oscillator in the BT formalism.

Finally, we would like to comment that the gauge invariant action (61) is not separated into the original action S_o and the new type of the WZ action S_{NWZ}. This is because X is nothing but the gauge degree of freedom.

4 Conclusion

In conclusion, we have applied the Batalin-Tyutin method, which converts the second class system into the first class, to the CSP theory and the quantum mechanical version of this theory. For the CSP case, we have shown that if we ignore the boundary term in action (31), the direct connection with the Lagrangian embedding of Stückelberg scalar can be made by explicitly evaluating the momentum integrals in the extended phase space partition function using the Faddeev-Popov-like gauges, and identifying the extra field ρ introduced in our Hamiltonian formalism with the conventional Stückelberg scalar needed to cancel the gauge anomaly due to the mass term. We have also obtained a new type of WZ action S_{NWZ} containing the WZ scalar λ, which is irrelevant to the gauge anomaly. Furthermore, we should also keep the boundary term S_B. Otherwise, we cannot reproduce the original first class system. Note that the Stückelberg scalar ρ is also included in S_{NWZ} in order to maintain the gauge invariance of the S_{NWZ} related to the CS effect in the action (31).

On the other hand, we have observed that the infrared limit of the CSP theory is precisely the gauge invariant CSP quantum mechanical model by using the BT formalism comparing with the symplectic formalism. Even though we can find a harmonic oscillator solution by solving the Hamilton equations of motion after applying the standard Dirac method, we have applied the symplectic method since it is more

intuitive when we directly find the generalized brackets from the Hamilton equation $f_{ij}\dot{\xi}^j = \frac{\partial}{\partial\xi^i}H(\xi)$ for the first-order system. In other words, if f_{ij} has an inverse, the Hamilton equation is easily obtained through $\dot{\xi}^i = \{\xi^i,\ H(\xi)\} = \{\xi^i,\ \xi^j\}\frac{\partial H(\xi)}{\partial\xi^j}$, where $\{\xi^i,\ \xi^j\}$ is the generalized bracket. Furthermore, by applying the BT formalism, we have firstly realized the harmonic oscillator at the action level, which is manifestly gauge invariant in the CSP quantum mechanical model.

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